ON THE CHARACTERIZATION OF POLYNOMIALS AND RATIONAL FUNCTIONS USING DIVIDED DIFFERENCES

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Abstract. In this paper we present two conjectures about the characterization of functions by conditions on their divided differences. To analyze the conjectures and prove some results, we recall some facts about the Hermite interpolation problem including the computation of divided differences for positive and negative powers of x.

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REFERENCES

[1] J. ACZÉL, *A mean value property of the derivative of quadratic polynomials – without mean values and derivatives*, Mathematics Magazine, 58:1 (1985), 42–45.

[2] J. ACZÉL, *Problem 296: Solution of $f(x) - g(x) = (x - y)h(x + y)$*, Two Year College Mathematical Journal, 18:4 (1987), 344–345.

[3] J. ACZÉL AND M. KUCZMA, *On two mean value properties and functional equations associated with them*, Aequationes Mathematicae, 38:2–3 (1989), 216–235.

[4] K. M. ANDERSEN, *A characterization of polynomials*, Mathematics Magazine, 69:2 (1996), 137–142.

[5] D. F. BAILEY, *A mean value property of cubic polynomials – without mean values*, Mathematics Magazine, 65:2 (1992), 123–124.

[6] D. F. BAILEY AND G. J. FIX, *A generalization of the mean value theorem*, Appl. Math. Lett., 1:4 (1988), 327–330.

[7] S. D. CONTE AND C. DE BOOR, *Elementary Numerical Analysis*, McGraw-Hill, New York, 1965.

[8] J. K. CHUNG AND P. K. SAHOO, *Characterizations of polynomials with mean value properties*, Appl. Math. Lett., 6:3 (1993), 97–100.

[9] G. E. CROSS AND P. L. KANNAPPAN, *A functional identity characterizing polynomials*, Aequationes Mathematicae, 34:2–3 (1987), 147–152.

[10] M. CROUZEIX AND A. L. MIGNOT, *Analyse numérique des équations différentielles*, Masson, Paris, 1984.

[11] P. J. DAVIES, *Interpolation & Approximation*, Dover, New York, 1963.

[12] R. O. DAVIES AND G. ROUSSEAU, *A divided difference characterization of polynomials over a general field*, Aequationes Mathematicae, 55:1–2 (1998), 73–78.

[13] C. DE BOOR, *A Practical Guide to Splines*, Springer-Verlag, New York, 1978.

[14] C. DE BOOR, *Divided Differences*, Surveys in Approximation Theory, 1 (2005), 46–69.

[15] E. DEEBA AND P. SIMEONOV, *Characterization of polynomials using divided differences*, Mathematics Magazine, 76:1 (2003), 61–66.

[16] D. E. DOBBS, *Average Values and Linear Functions*, Two Year College Mathematical Journal, 16:2 (1985), 132–135.

[17] H. H. DOWNING, *Note on the law of the mean*, The American Mathematical Monthly, 27:12 (1920), 467–469.

[18] F. DUBEAU, *On Hermite interpolation and divided differences*, Surveys in Mathematics and its Applications, 15 (2020), 257–277.

[19] S. Haruki, *A property of quadratic polynomials*, The American Mathematical Monthly, 86:7 (1979), 577–579.

[20] P. L. Kannappan and P. K. Sahoo, *Characterization of polynomials and divided difference*, Proc. Indian Acad. Sci. (Math. Sci.), 105 (1995), 287–290.

[21] M. S. Klamkin and D. J. Newman, *On Some Inverse Problems in Potential Theory*, Quarterly Applied Math., 26 (1968), 277–280.

[22] E. T. Y. Lee, *A remark on divided differences*, The American Mathematical Monthly, 96:7 (1989), 618–622.

[23] M. Z. Nashed, *A Functional Equation Which Characterizes Polynomials with Applications to Uniqueness*, Applicable Analysis, 5 (1975), 55–63.

[24] T. L. Saaty, *Modern Nonlinear Equations*, McGraw-Hill, New York, 1967.

[25] J. Schwaiger, *On a characterization of polynomials by divided differences*, Aequationes Mathematicae, 48:2–3 (1994), 317–323.

[26] B. Shawyer and B. Watson, *Problem 312: Mean value theorem for polynomials*, Two Year College Mathematical Journal, 18:4 (1987), 344–345.

[27] P. Simeonov, *Characterization of a class of functions using divided differences*, Abstract and Applied Analysis, 5:2 (2000), 85–90.

[28] A. Spitzbart, *A generalization of Hermite's interpolation formula*, The American Mathematical Monthly, 67:1 (1960), 42–46.

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