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NONLINEAR DYNAMIC EQUATIONS ON TIME SCALES WITH IMPULSES AND NONLOCAL CONDITIONS

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Abstract. The purpose of this paper is to introduce more general results on the existence of solutions for nonlinear dynamic equations on time scales with impulses and nonlocal initial conditions. We establish the existence of solutions by applying a fixed point result due to O'Regan, while the uniqueness of solutions is obtained through the contraction mapping principle. Our results extend previous work in the literature and an example is discussed to illustrate the obtained results.

Mathematics subject classification (2010): 34N05, 34A12, 34A37, 39A12.
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