Journal of Classical Analysis
Volume 17, Number 1 (2021), 1–11
doi:10.7153/jca-2021-17-01

A POISSON LOGARITHMIC INTEGRAL FOR INTEGER ORDER POWERS n = 0, 1, 2, AND 3

SEÁN M. STEWART

Abstract. We give analytic expressions for the Poisson type logarithmic integral

$$\mathrm{Lp}_n(a) = \int_0^\pi \log^n(1-2a\cos x+a^2)\,dx,$$

for integer order powers $n = 0, 1, 2$, and 3. Here a is any real number. A generalisation of the integral for the $n = 2$ case is also given.

Mathematics subject classification (2010): Primary: 40C10; Secondary: 42A16.
Keywords and phrases: Poisson integral, log-sine integral, polylogarithm, Euler sums.

REFERENCES

[1] S.-D. POISSON, *Suite du mémoire sur les intégrales définies*, J. Éc. polytech. Math., Cahier 17 **10**, (1815), 612–631.

[2] CH. DELAUNAY, *Détermination de l'intégrale définie* $\int_0^\pi \log(1-2a\cos x+a^2)\,dx$, J. Math. Pures Appl. **3**, (1838), 355–356.

[3] D. BIERENS DE HAAN, *Exposé de la théorie, des propriétés, des formules de transformation, et des méthodes d'évaluation des intégrales définies*, C. G. van der Post, Amsterdam, 1862.

[4] H. GOODWIN, *On some definite integrals*, Cambridge Math. J., **3**, 16 (1843), 168–169.

[5] JOHANN AUGUST GRUNERT, *Ueber die neuesten Erfindungen in der Theorie der bestimmten Integrale*, Arch. Math. Phys., **4**, (1844), 113–126.

[6] HONGWEI CHEN, *Four ways to evaluate a Poisson integral*, Math. Mag., **75**, 4 (2002), 290–294.

[7] I. S. GRADSHTEYN AND I. M. RYZHIK (Eds), *Table of Integrals, Series, and Products*, Eighth Edition, Academic Press, Amsterdam, 2015.

[8] WOLFGANG GRÖBNER AND NIKOLAUS HOFREITER (Eds), *Integraltafel: Zweiter Teil, Bestimmte Integrale*, Fifth Edition, Springer-Verlag, Wien, 1973.

[9] F. BOWMAN, *Note on the integral* $\int_0^{\frac{1}{2}\pi} (\log\sin\theta)^n\,d\theta$, J. London Math. Soc., **22**, 3 (1947), 172–173.

[10] L. LEWIN, *On the evaluation of log-sine integrals*, Math. Gaz., **42**, 340 (1958), 125–128.

[11] M. G. BEUMER, *Some special integrals*, Amer. Math. Monthly, **68**, 7 (1961), 645–647.

[12] A. A. JAGERS AND E. M. J. NIESSEN, *Problem 92-4: A definite integral arising in Ohmic dissipation*, SIAM Rev., **34**, 1 (1992), 120, 126.

[13] K. S. KÖLBIG, *On the integral* $\int_0^{\pi/2} \log^n\cos x\log^p\sin x\,dx$, Math. Comp., **40**, 162 (1983), 565–570.

[14] LEONARD LEWIN, *Polylogarithms and Associated Functions*, North Holland, New York, 1981.