APPROXIMATION BY INTERPOLATION: THE CHEBYSHEV NODES

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Abstract. In this paper, we first revisit the well-known result stating that the Hermite interpolation polynomials of a function f continuous on $[-1,1]$, with the zeros of the Chebyshev polynomials of the first kind as nodes, converge uniformly to f on $[-1,1]$. Then we extend this result to obtain the uniform convergence of the Hermite interpolation polynomials, with the nodes taken as the zeros of the Chebyshev polynomials of the second, third and fourth kind, not on the interval $[-1,1]$ but rather on the intervals $[-\frac{2\sqrt{2}}{3}, \frac{2\sqrt{2}}{3}]$, $[-\frac{\sqrt{3}}{2}, 1]$, $[-1, \frac{\sqrt{3}}{2}]$, respectively.

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