

## APPLICATIONS OF BRIOT–BOUQUET DIFFERENTIAL SUBORDINATION

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*Abstract.* Sharp estimates on  $\beta$  in Briot-Bouquet differential subordination

$$p(z) + \beta zp'(z)/p(z) \prec h(z)$$

are obtained so that its solution  $p$  is subordinate to some specific Carathéodory functions. As an application, the estimates on  $\beta$  are obtained so that the integral operator  $\beta^{-1} \int_0^z f^{1/\beta}(t)t^{-1} dt$  maps the class of starlike functions  $f$  satisfying  $zf'(z)/f(z) \prec 1+z$  to various subclasses of class of starlike functions. Further, a sufficient condition is established for parabolic starlikeness.

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