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FOURIER TRANSFORM INVERSION IN THE ALEXIEWICZ NORM

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Abstract. If $f\in L^1(\mathbb{R})$ it is proved that $\lim_{S\to\infty}\|f-f*D_S\|=0$, where $D_S(x)=\sin(Sx)/(\pi x)$ is the Dirichlet kernel and $\|f\|=\sup_{\alpha<\beta}|\int_\alpha^\beta f(x)\,dx|$ is the Alexiewicz norm. This gives a symmetric inversion of the Fourier transform on the real line. An asymmetric inversion is also proved. The results also hold for a measure given by dF where F is a continuous function of bounded variation. Such measures need not be absolutely continuous with respect to Lebesgue measure. An example shows there is $f\in L^1(\mathbb{R})$ such that $\lim_{S\to\infty}\|f-f*D_S\|_1\neq 0$.

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