ON THE LOCATION OF ZEROS OF QUASI–ORTHOGONAL POLYNOMIALS WITH APPLICATIONS TO SOME REAL SELF–RECIPROCAL POLYNOMIALS

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Abstract. In this paper, we present new results on the location of zeros of some classes of quasi-orthogonal polynomials. From the Chebyshev polynomials, we obtain some classes of real self-reciprocal polynomials, and investigate the location and monotonicity of their zeros.

REFERENCES

[1] M. ALFARO, F. MARCELLÁN, A. PEÑA AND M. L. REZOLA, *When do linear combinations of orthogonal polynomials yield new sequences of orthogonal polynomials?*, J. Comput. Appl. Math. **233** (6) (2010), 1446–1452.

[2] A. F. BEARDON, K. A. DRIVER, *The zeros of linear combinations of orthogonal polynomials*, J. Approx. Theory **137** (2005), 179–186.

[3] E. BERRIOCHOA, A. CACHAFEIRO, J. M. GARCÌA–AMOR, *A characterization of the four Chebyshev orthogonal families*, Int. J. Math. Sci. **13** (2005), 2071–2079.

[4] V. BOTTA, C. F. BRACCIALI, J. A. PEREIRA, *Some properties of classes of real self–reciprocal polynomials*, J. Math. Anal. Appl. **433** (2016), 1290–1304.

[5] C. F. BRACCIALI, D. K. DIMITROV AND A. SRI RANGA, *Chain sequences and symmetric generalized orthogonal polynomials*, J. Comput. Appl. Math. **143** (2002), 95–106.

[6] C. F. BRACCIALI, F. MARCELLÁN AND S. VARMA, *Orthogonality of Quasi-Orthogonal Polynomials*, Filomat **32:20** (2018), 6935–6977.

[7] C. F. BRACCIALI, A. SRI RANGA, A. SWAMINATHAN, *Para-orthogonal polynomials on the unit circle satisfying three term recurrence formulas*, Appl. Numer. Math. **109** (2016), 19–40.

[8] C. F. BRACCIALI, XIN LI, A. SRI RANGA, *Real orthogonal polynomials in frequency analysis*, Math. Comp. **74** (2004), 341–362.

[9] C. BREZINSKI, K. A. DRIVER AND M. REDIVO–ZAGLIA, *Quasi-orthogonality with applications to some families of classical orthogonal polynomials*, Appl. Numer. Math. **48** (2004), 157–168.

[10] M. J. CANTERO, L. MORAL, L. VELÁZQUEZ, *Measures and para-orthogonal polynomials on the unit circle*, East J. Approx. **8** (2002), 447–464.

[11] T. S. CHIHARA, *An Introduction to Orthogonal Polynomials*, Gordon and Breach, New York, 1978.

[12] T. S. CHIHARA, *On quasi-orthogonal polynomials*, Proc. Amer. Math. Soc. **8** (1957), 765–767.

[13] M. S. COSTA, H. M. FELIX, A. SRI RANGA, *Orthogonal polynomials on the unit circle and chain sequences*, J. Approx. Theory **173** (2013), 14–32.

[14] L. DARUIS, O. NJÅSTAD, W. VAN ASSCHE, *Para-orthogonal polynomials in frequency analysis*, Rocky Mountain J. Math. **33** (2003), 629–645.

[15] P. DELSARTE, Y. GENIN, *The split Levinson algorithm*, IEEE Trans. Acoust. Speech Signal Process. **34** (1986), 470–478.

[16] K. DRIVER, K. JORDAAN, *Zeros of a Quasi-Orthogonal Jacobi Polynomials*, SIGMA Symmetry Integrability Geom. Methods Appl. **12** (2016), 042, 13 pages.

[17] K. DRIVER, M. E. MULDOON, *Zeros of quasi-orthogonal ultraspherical polynomimals*, Indag. Math. (N.S.) **27** 4 (2016), 930–944.

[18] L. FÉJER, *Mechanische Quadraturen mit positiven Cotesschen Zahlen*, Math. Z. **37** (1933), 287–309.

[19] E. GHATE, E. HIRONAKA, *The arithmetic and geometry of Salem numbers*, Bull. Amer. Math. Soc. **38** (2001), 293–314.

[20] L. GOLINSKII, *Quadrature formula and zeros of para-orthogonal polynomials on the unit circle*, Acta Math. Hungar. **96** (2002), 169–186.

[21] D. GÓMEZ-ULLATE, F. MARCELLÁN, R. MILSON, *Asymptotic and interlacing properties of zeros of exceptional Jacobi and Laguerre polynomials*, J. Math. Anal. Appl. **399** (2) (2013), 480–495.

[22] Z. GRINSHPUN, *Special linear combinations of orthogonal polynomials*, J. Math. Anal. Appl. **299** (2004), 1–18.

[23] C. L. HO, R. SASAKI, *Zeros of the Exceptional Laguerre and Jacobi polynomials*, ISRN Math. Physics (2012), 27 pages, https://doi.org/10.5402/2012/920475.

[24] W. B. JONES, O. NJÅSTAD, W. THRON, *Moment theory, orthogonal polynomials, quadrature, and continued fractions associated with the unit circle*, Bull. Lond. Math. Soc. **21** (2) (1989), 113–152.

[25] H. JOULAK, *A contribution to quasi-orthogonal polynomials and associated polynomials*, Appl. Numer. Math. **54** (2005), 65–78.

[26] D. JOYNER, *Zeros of some self-reciprocal polynomials*, in: T.D. Andrews, et al. (Eds.), Excursions in Harmonic Analysis, Vol. 1, Springer (2013), 329–348.

[27] D. JOYNER, J.-L. KIM, *Selected unsolved problems in coding theory*, Appl. Numer. Harmon. Anal., Birkhäuser (2011).

[28] S. H. KIM AND C. W. PARK, *On the zeros of certain self-reciprocal polynomials*, J. Math. Anal. Appl. **339** (2008), 240–247.

[29] P. LAKATOS, *On zeros of reciprocal polynomials*, Publ. Math. Debrecen **61** (2002), 645–661.

[30] L. LOSONCZI, *On reciprocal polynomials with zeros of modulus one*, Math. Inequal. Appl. **9** (2006), 286–298.

[31] G. V. MILOVANOVIĆ, D. S. MITRINOVIĆ, TH. M. RASSIAS, *Topics in polynomials: extremal problems, inequalities, zeros*, World Scientific (1994).

[32] M. RIESZ, *Sur le problème des moments*, Troisième Note, Ark. Mat. Fys. **17** (1923), 1–52.

[33] J. A. SHOHAT, *On mechanical quadratures, in particular, with positive coefficients*, Trans. Amer. Math. Soc. **42** (1937), 461–496.

[34] G. SZEGÖ, *Orthogonal Polynomials*, Amer. Math. Soc. Colloq. Publ. Vol **23**. Amer. Math. Soc. Providence, RI 1975. Fourth Edition.

[35] R. S. VIEIRA, *Polynomials with symmetric zeros*, IntechOpen (2019). Available on https://doi.org/10.5772/intechopen.82728, doi:10.5772/intechopen.82728.

[36] R. S. VIEIRA, V. BOTTA, *Orthogonal polynomials and Möbius transformations*, Comput. Appl. Math. **40** (2021) no. 6, paper 181.

[37] Y. XU, *A characterization of positive quadrature formula*, Math. Comp. **62** 206 (1994), 703–718.

[38] A. ZHEDANOV, *Biorthogonal Rational Functions and the Generalized Eigenvalue Problem*, J. Approx. Theory **101** (1999), pp. 303–329.

[39] A. ZHEDANOV, *On some classes of polynomials orthogonal on arcs of the nit circle connected with symmetric orthogonal polynomials on an interval*, J. Approx. Theory, **94** (1998), pp. 73–106.

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