Journal of Classical Analysis
Volume 20, Number 1 (2022), 23–33
doi:10.7153/jca-2022-20-03

REFINED NORMAL APPROXIMATIONS FOR THE STUDENT DISTRIBUTION

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Abstract. In this paper, we develop a local limit theorem for the Student distribution. We use it to improve the normal approximation of the Student survival function given in [23] and to derive asymptotic bounds for the corresponding maximal errors at four levels of approximation. As a corollary, approximations for the percentage points (or quantiles) of the Student distribution are obtained in terms of the percentage points of the standard normal distribution.

Mathematics subject classification (2020): Primary 62E20; Secondary 60F99.
Keywords and phrases: Asymptotic statistics, local limit theorem, Gaussian approximation, normal approximation, Student distribution, error bound, survival function, percentage point, quantiles, detection theory.

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