

COEFFICIENT PROBLEMS OF A CLASS OF q -STARLIKE FUNCTIONS ASSOCIATED WITH q -ANALOGUE OF AL-BOUDI-AL-QAHTANI INTEGRAL OPERATOR AND NEPHROID DOMAIN

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Abstract. This investigation is on a set $SN_q^*(n, \tau; \eta)$ of q -starlike functions defined by using a newly defined q -analogue of Al-Oboudi-Al-Qahtani integral operator along with subordination and nephroid domain. This new q -operator generalizes some known integral operators. Results such as coefficient bounds, Fekete-Szegő problem (for real and complex parameters) and bounds of some Hankel determinants are presented. Our results generalize some known and new ones.

Mathematics subject classification (2020): 30C45, 30C50.

Keywords and phrases: Starlike function, nephroid domain, Al-Oboudi-Al-Qahtani integral operator, q -calculus, convolution, subordination, Fekete-Szegő functional, coefficient bounds, Hankel determinants.

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