

FEKETE–SZEGŐ INEQUALITY FOR CLASSES OF ANALYTIC FUNCTIONS CONNECTED WITH THE (p, q) -DERIVATIVE

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Abstract. In this paper, we introduce the new classes $\mathcal{S}_{\lambda, p, q}^*(\eta, \zeta, \varphi)$ and $\mathcal{C}_{\lambda, p, q}(\eta, \zeta, \varphi)$ of analytic functions in the open unit disc, by using the (p, q) -derivative, which are a generalization of the known starlike and convex functions of complex order, respectively. Our aim for these classes is to investigate the Fekete-Szegő inequalities. The various results, which are presented in this paper, would generalize those in related works of several earlier authors.

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