

NEW CONVERGENCE DEFINITIONS FOR  
DOUBLE SEQUENCES IN  $g$ -METRIC SPACES

MEHMET GÜRDAL, ÖMER KIŞI\* AND SAIME KOLANCI

**Abstract.** In this paper, we define  $g$ -convergence and  $g$ -Cauchy of double sequences in  $g$ -metric spaces. Also we prove that  $g$ -limit is unique and every  $g$ -convergent double sequence is a  $g$ -Cauchy sequence. Additionally  $g$ -statistical convergence of double sequences is introduced and the theorem giving the relationship between statistical convergence and strongly Cesàro summability in a  $g$ -metric space is demonstrated. Further, we put forward the notations of  $g$ -lacunary statistical convergence and  $g$ -strongly lacunary convergence of double sequences and we also present some inclusion theorems.

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