

COMPLETE ASYMPTOTIC EXPANSION OF
THE COMPOUND MEANS WITH APPLICATIONS

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Abstract. We present a complete asymptotic expansion of the compound mean $M \otimes N$ of two symmetric homogeneous means M and N and derive an efficient algorithm for computing coefficients in this expansion. This new approach is applied to obtain a simple formula for computing coefficients in the expansion of the arithmetic-geometric mean. We also give some other applications to the compounds of the classical means.

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