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OCTONION WINDOWED LINEAR CANONICAL TRANSFORM

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Abstract. The Linear Canonical Transform (LCT) is a mathematical transform that generalizes several well-known transforms, including the Fourier transform, the fractional Fourier transform, and the Fresnel transform. It provides a unified framework for understanding and representing a wide range of linear and linear-like transforms, allowing for the analysis and manipulation of signals in various domains. Recently, Gao et al. extended the notion of LCT to octonion domains and showed its efficacy in precisely representing the non-transient octonion-valued signals. However, the octonion LCT exhibits limitations in effectively localizing the frequency characteristics of non-transient octonion-valued signals. As such, it is imperative to introduce the Octonion Windowed Linear Canonical Transform (OCWLCT) and explore its fundamental characteristics. We delve into the inversion formula and the orthogonality relation for the one-dimensional OCWLCT. Additionally, we derive the inversion formula for the three-dimensional Octonion Windowed Linear Canonical Transform (OCWLCT).

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