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FUNCTIONAL DEUTSCH UNCERTAINTY PRINCIPLE

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Abstract. Entropic uncertainty principle for finite dimensional Hilbert spaces (known as Deutsch uncertainty) obtained by Deutsch *[Phys. Rev. Lett., 1983]* is a foundational result in Mathematics and Physics. We derive the Deutsch uncertainty principle for finite dimensional Banach space and its dual. Our main tool is the notion of Parseval p-frames for Banach spaces. Using the celebrated Buzano inequality in Hilbert spaces, we show that our result reduces to the Deutsch uncertainty principle for Hilbert spaces.

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