ON THE ZEROS OF QUATERNIONIC
POLYNOMIAL WITH RESTRICTED COEFFICIENTS

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Abstract. Location of the zeros of regular polynomial of a quaternionic variable with quaternionic coefficients is addressed in this study. We derive new bounds for the zeros of such polynomials by virtue of the structure of the zero sets in the newly developed theory of polynomials of a quaternionic variable. We will generalize some recently proven results concerning the distribution of zeros of a quaternionic polynomial with restricted coefficients.

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