

## A SERIES OF RAMANUJAN, TWO-TERM DILOGARITHM IDENTITIES AND SOME LUCAS SERIES

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*Abstract.* We study an elementary series that can be considered a relative of a series studied by Ramanujan in Part 1 of his Lost Notebooks. We derive a closed form for this series in terms of the inverse hyperbolic arctangent and the polylogarithm. Special cases will follow in terms of the Riemann zeta and the alternating Riemann zeta function. In addition, some trigonometric series will be expressed in terms of the Clausen functions. Finally, a range of new two-term dilogarithm identities will be proved and some difficult series involving Lucas numbers will be evaluated in closed form.

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