ON LACUNARY $\mathscr{I}$–INVARIANT CONVERGENCE OF SEQUENCES IN QUATERNION–VALUED GENERALIZED METRIC SPACES

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Abstract. In this study, we explore the concept of lacunary $\mathscr{I}_\sigma$-convergence of sequences and analyze the relationships between this new convergence concept and the notions of lacunary invariant summability, lacunary strongly s-invariant summability, and lacunary σ-statistical convergence, all of which are defined within quaternion-valued generalized metric spaces. Additionally, our paper aims to introduce the concepts of lacunary $\mathscr{I}_\sigma^*$-convergence in quaternion-valued generalized metric spaces. We then establish the equivalence between lacunary $\mathscr{I}_\sigma$-convergence and lacunary $\mathscr{I}_\sigma^*$-convergence by providing the definition of property (AP). Furthermore, we introduce lacunary $\mathscr{I}_\sigma$-Cauchy and lacunary $\mathscr{I}_\sigma^*$-Cauchy sequences, adapting classical theorems to quaternion-valued generalized metric spaces.

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