

A NOTE ON A GENERALISATION OF A DEFINITE INTEGRAL
INVOLVING THE BESSEL FUNCTION OF THE FIRST KIND

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Abstract. We consider a generalisation of a definite integral involving the Bessel function of the first kind. It is shown that this integral can be expressed in terms of the Fox-Wright function ${}_p\Psi_q(z)$ of one variable. Some consequences of this representation are explored by suitable choice of parameters. Further, we compute the range of numerical approximation values of the Ramanujan's cosine integral $\phi_C(m, n)$ and sine integral $\phi_S(m, n)$ for distinct values of m and n by Wolfram Mathematica software. In addition, two closed-form evaluations of infinite series of the Fox-Wright function are deduced and these sums have been verified numerically using Mathematica.

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