

REDUCTION AND SUMMATION FORMULAS FOR APPELL FUNCTIONS IN THE SPIRIT OF BURCHNALL–CHAUNDY

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Abstract. We find some reduction and summation formulas for multiple hypergeometric functions by using Burchnell-Chaundy expansions. The proofs use Legendre duplication, Kummer's second summation formula, Gauss summation formula and Whipple transformation. Many times we simply put a hypergeometric function argument equal to zero in order to cancel this second factor. We conclude by proving a q -analogue of a reduction formula for the third q -Appell function.

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