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HANKEL DETERMINANTS FOR LOGARITHMIC AND LOGARITHMIC INVERSE COEFFICIENTS FOR THE CLASS $\mathscr{U}(\lambda)$

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Abstract. Let $\mathscr{U}(\lambda)$ be the class of analytic functions f in the open unit disc $\mathbb{D}$ with the normalization $f(0)=0$ and $f'(0)=1$ satisfying $\left|\left(\frac{z}{f(z)}\right)^2 f'(z)-1\right|<\lambda$ for $0<\lambda\leqslant 1$. In this paper, we obtain the sharp bounds of the second Hankel determinant of logarithmic and logarithmic inverse coefficients, and the sharp bounds of the first three logarithmic coefficients for $f\in\mathscr{U}(\lambda)$.

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