Journal of Classical Analysis
Volume 26, Number 1 (2025), 47–61
doi:10.7153/jca-2025-26-04

BOUNDED AND UNBOUNDED BERGMAN TYPE PROJECTIONS ON THE BLOCH SPACE

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Abstract. We prove that harmonic Bergman projection is unbounded on the Bloch space $\mathscr{B}$ over the unit ball in $\mathbb{R}^n$. Another family of Bergman type operators is found whose members continuously project the Bloch space of smooth functions $\mathscr{B}$ onto its harmonic subspace $h\mathscr{B}$. A generalization with more general indices is also given. Our method is mainly based on the techniques of a modified fractional integro-differentiation and two-sided estimates of the reproducing kernels and integrals.

Mathematics subject classification (2020): 31B10, 26A33, 42B35, 46E15.
Keywords and phrases: Bloch space, harmonic Bergman projection, fractional derivative, Poisson-Bergman kernel.

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