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A MOLECULAR DECOMPOSITION FOR $H^p(\mathbb{Z}^n)$

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Abstract. In this work, for the range $\frac{n-1}{n} < p \leqslant 1$, we give a molecular reconstruction theorem for $H^p(\mathbb{Z}^n)$. As an application of this result and the atomic decomposition developed by S. Boza and M. Carro in [Proc. R. Soc. Edinb., 132 A (1) (2002), 25–43], we prove that the discrete Riesz potential I_α defined on $\mathbb{Z}^n$ is a bounded operator $H^p(\mathbb{Z}^n) \to H^q(\mathbb{Z}^n)$ for $\frac{n-1}{n} < p < \frac{n}{\alpha}$ and $\frac{1}{q} = \frac{1}{p} - \frac{\alpha}{n}$, where $0 < \alpha < n$.

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