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NONNEGATIVE SOLUTIONS OF DISCRETE SECOND–ORDER UNDAMPED STURM—LIOUVILLE BOUNDARY VALUE PROBLEMS

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Abstract. This article considers a second-order undamped difference equation satisfying the Sturm–Liouville boundary conditions. We apply suitable fixed point theorems to establish sufficient conditions for the existence of nonnegative solutions to the considered problem. We also provide two examples to illustrate the applicability of established results.

Though there have been many works on the study of nonnegative solutions to the considered problem, most of the works used the application of the Guo–Krasnoselskii fixed point theorem. This work's novelty lies in applying new fixed point theorems that yield multiple fixed points in conical shells to the considered problem. This work improves some previous results and reports some new results.

Mathematics subject classification (2020): 39A05, 39A12, 39A27, 39A70.

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