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$\mathscr{A}_f$–STATISTICAL CONVERGENCE AND $\mathscr{A}_f$–STATISTICAL BOUNDEDNESS BY MODULI IN $\mathscr{A}$–METRIC SPACES

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Abstract. In recent years, researchers have been intensely studying the important concepts of summability theory in various generalizations of metric spaces. This has motivated us to investigate the concepts of statistical convergence, statistical boundedness, and Cesàro summability, which respect the modulus function, in $\mathscr{A}$-metric spaces, a generalization of usual metric spaces. In this study, we introduce $\mathscr{A}_f$-statistical convergence, $\mathscr{A}_f$-statistical boundedness, and $\mathscr{A}_f$-strong Cesàro summability with respect to a modulus for sequences in $\mathscr{A}$-metric spaces. We explore the relationships between $\mathscr{A}_f$-statistically convergent sequences and $\mathscr{A}_f$-statistically bounded sequences with respect to a modulus. Additionally, we investigate the connections between the set of $\mathscr{A}_f$-statistically convergent sequences and the set of $\mathscr{A}_f$-strongly Cesàro summable sequences, defined using the modulus function.

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