

## THE ANGER DIFFERENCE EQUATION

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*Abstract.* A nonhomogeneous Bessel difference equation, dubbed the Anger difference equation, is solved and properties of its solution are investigated. Numerical evidence that the solution is oscillatory is presented and difficulties in finding a proof for it are discussed.

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### REFERENCES

- [1] RAVI P. AGARWAL, *Discrete oscillation theory*, vol. 1, Hindawi Publishing Corporation, 2005.
- [2] C. T. ANGER, *Neueste Schr. d. Naturf. d. Ges. i. Danzig*, 5: 1–29, 1855.
- [3] EMIL ARTIN, *The Gamma function*, Courier Dover Publications, 2015.
- [4] AMAR BAŠIĆ, LEJLA SMAJLOVIĆ AND ZENAN ŠABANAC, *Discrete Bessel functions and discrete wave equation*, *Results in Mathematics*, **79** (5): 216, 2024.
- [5] M. BOHNER AND A. PETERSON, *Dynamic equations on time scales: An introduction with applications*, Birkhäuser Boston, Inc., Boston, MA, 2001.
- [6] MARTIN BOHNER AND TOM CUCHTA, *The Bessel difference equation*, *Proc. Amer. Math. Soc.*, **145** (4): 1567–1580, 2017.
- [7] MARTIN BOHNER AND TOM CUCHTA, *The generalized hypergeometric difference equation*, *Demonstratio Mathematica*, **51** (1): 62–75, 2018.
- [8] MARTIN BOHNER AND CHRISTOPHER TISDELL, *Oscillation and nonoscillation of forced second order dynamic equations*, *Pacific Journal of Mathematics*, **230** (1): 59–71, 2007.
- [9] DIEGO CARATELLI, *Incomplete Anger–Weber Functions: A Class of Special Functions for Electromagnetics*, *Radio Science*, **54** (4): 331–348, 2019.
- [10] TOM CUCHTA, DAVID GROW AND NICK WINTZ, *Discrete matrix hypergeometric functions*, *Journal of Mathematical Analysis and Applications*, **518** (2): 126716, 2023.
- [11] KE FENG, CHANGHAI YU, JIANGSHENG LIU, WENTAO WANG, YE TIAN, ZHIJUN ZHANG, RONG QI, MING FANG, JIAQI LIU, ZHIYONG QIN, YING WU, YU CHEN, LINTONG KE, CHENG WANG AND RUXIN LI, *Coherent X-ray source generation with off-resonance laser modulation*, *Optics Express* **26** (15): 19067, 2018.
- [12] I. S. GRADŠTEJN AND I. M. RYŽIK, *Table of integrals, series, and products*, 2000.
- [13] NAHOMI KAN AND KIYOSHI SHIRAIISHI, *Discrete time heat kernel and UV modified propagators with dimensional deconstruction*, *J. Phys. A Math. Theor.*, **56** (24): 245401, 2023.
- [14] WALTER G. KELLEY AND ALLAN C. PETERSON, *Difference equations: an introduction with applications*, Academic press, 2001.
- [15] FRANK OLVER, DANIEL LOZIER, RONALD BOISVERT AND CHARLES CLARK, *The NIST Handbook of Mathematical Functions*, Cambridge University Press, New York, NY, 2010-05-12 00:05:00, 2010.
- [16] ANATOLIĬ PLATONOVICH PRUDNIKOV, YURY ALEKSANDROVICH BRYCHKOV AND OLEG IGOREVICH MARICHEV, *Integrals and series: direct Laplace transforms*, vol. 4, CRC Press, 1986.
- [17] ANTONÍN SLAVÍK, *Discrete Bessel functions and partial difference equations*, *J. Difference Equ. Appl.*, **24** (3): 425–437, 2018.
- [18] ANTONÍN SLAVÍK, *Asymptotic behavior of solutions to the semidiscrete diffusion equation*, *Appl. Math. Lett.*, **106**: 106392, 7, 2020.

- [19] NICOLA TEDESCHI AND FABRIZIO FREZZA, *Analytic solution for the reflection of cylindrical wave at planar interfaces*, in: 2014 XXXIth URSI General Assembly and Scientific Symposium (URSI GASS), IEEE, 2014.
- [20] AHMET YANTIR, BURCU SILINDIR YANTIR AND ZEHRA TUNCER, *Bessel equation and Bessel function on  $\mathbb{T}_{(q,h)}$* , Turkish J. Math., **46** (8): 3300–3322, 2022.