Journal of Classical Analysis
Volume 27, Number 2 (2026), 197–208
doi:10.7153/jca-2026-27-14

ON COMPLEX INTERPOLATION IN VARIABLE LEBESGUE SPACES WITH MIXED NORM

ROVSHAN BANDALIYEV* AND MEHRALY ALIYEV

Abstract. In this paper, we prove a multiplicative interpolation inequality in variable Lebesgue spaces with mixed norm. By the method of complex interpolation, we define an interpolation spaces between variable Lebesgue spaces with mixed norm. As an application of the main result we establish an analog of Riesz-Thorin interpolation theorem in variable Lebesgue spaces with mixed norm.

Mathematics subject classification (2020): 46B70, 46E30.
Keywords and phrases: Variable Lebesgue space with mixed norm, interpolation inequality, interpolation space, complex interpolation method, Riesz-Thorin interpolation theorem, embedding theorems.

REFERENCES

[1] E. ACERBI AND G. MINGIONE, *Regularity results for electrorheological fluids: the stationary case*, C. R. Math. Acad. Sci. Paris **334**, 9 (2002), 817–822.
[2] E. ACERBI AND G. MINGIONE, *Gradient estimates for the $p(x)$-Laplacean system*, J. Reine Angew. Math. **584**, (2005), 117–148.
[3] D. R. ALIYEVA AND R. A. BANDALIYEV, *On sharp constant in generalized Minkowski inequality on variable Lebesgue spaces*, Trans. Natl. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci. Mathematics **42**, 4 (2022), 22–28.
[4] A. N. ARTYUSHIN, *Function spaces of $L_{p(\cdot)}\left(L_{q(\cdot)}\right)$-type and embedding theorems for spaces with variable smoothness*, Sib. Math. J. **66**, 1 (2025), 1–15.
[5] R. A. BANDALIEV AND M. M. ABBASOVA, *On an inequality and $p(x)$-mean continuity in the variable Lebesgue space with mixed norm*, Trans. Azerb. Natl. Acad. Sci. Ser. Phys.-Tech. Math. Sci. **26**, 7 (2006), 47–56.
[6] R. A. BANDALIEV, *On an inequality in Lebesgue space with mixed norm and with variable summability exponent*, Math. Notes **84**, 3 (2008), 303–313, corrigendum in Math. Notes **99**, 2 (2016), 340–341.
[7] R. A. BANDALIEV, *Embedding beetween variable exponent Lebesgue spaces with measures*, Azerb. J. Math. **2**, 1 (2012), 119–126.
[8] R. A. BANDALIEV, *Applications of multidimensional Hardy operator and its connection with a certain nonlinear differential equation in weighted variable Lebesgue spaces*, Ann. Funct. Anal. **4**, 2 (2013), 118–130.
[9] A. BENEDEK AND R. PANZONE, *The spaces L^p, with mixed norm*, Duke Math. J. **28**, 3 (1961), 301–324.
[10] J. BERGH AND J. LÖFSTRÖM, *Interpolation spaces. An introduction*, Grundlehren der Mathematischen Wissenschaften **223**, Springer-Verlag, Berlin Heidelberg, New York, (1976).
[11] P. BLOMGREN, T. CHAN, P. MULET AND C. K. WONG, *Total variation image restoration: numerical methods and extensions*, in Proceedings of the 1997 IEEE International Conference on Image Processing **3**, (1997), 384–387.
[12] G. BUTAKIN, E. PIŞKIN AND E. ÇELIK, *Blowup and global solutions of a fourth-order parabolic equation with variable exponent logarithmic nonlinearity*, J. Funct. Spaces **2024**, 1 (2024), 2847533, 1–9.
[13] B. ÇEKIÇ, A. V. KALININ, R. A. MASHIYEV AND M. AVCI, *$L^{p(x)}$-estimates for vector fields and some applications to magnetostatics problems*, J. Math. Anal. Appl. **389**, 2 (2012), 838–851.

Paper JCA-27-14

[14] D. CRUZ-URIBE AND A. FIORENZA, *Variable Lebesgue spaces. Foundations and harmonic analysis*, Applied and Numerical Harmonic Analysis, Birkhäuser/Springer, Heidelberg, New York, Dordrecht, London, (2013).
[15] L. DIENING, P. HARJULEHTO, P. HÄSTÖ AND M. RŮŽIČKA, *Lebesgue and Sobolev spaces with variable exponents*, Lecture Notes in Math. **2017**, Springer-Verlag, Heydelberg, Berlin, (2011).
[16] L. DIENING AND M. RŮŽIČKA, *Calderón-Zygmund operators on generalized Lebesgue spaces $L^{p(\cdot)}$ and problems related to fluid dynamics*, J. Reine Angew. Math. **563**, (2003), 197–220.
[17] L. DIENING AND M. RŮŽIČKA, *Integral operators on the halfspace in generalized Lebesgue spaces $L^{p(\cdot)}$*, I, J. Math. Anal. Appl. **298**, 2 (2004), 559–571.
[18] L. DIENING AND M. RŮŽIČKA, *Integral operators on the halfspace in generalized Lebesgue spaces $L^{p(\cdot)}$*, II, J. Math. Anal. Appl. **298**, 2 (2004), 572–588.
[19] V. KOKILASHVILI, A. MESKHI, H. RAFEIRO AND S. SAMKO, *Integral Operators in Non-standard Function Spaces. I, Variable exponent Lebesgue and amalgam spaces*, Operator Theory: Advances and Applications **248**, Birkhäuser, Springer, Basel, (2016).
[20] V. KOKILASHVILI, A. MESKHI, H. RAFEIRO AND S. SAMKO, *Integral Operators in Non-standard Function Spaces. 2, Variable exponent Hölder, Morrey-Campanato and grand spaces*, Birkhäuser, Springer, Basel, 2016.
[21] O. KOVACIK AND J. RAKOSNIK, *On spaces $L^{p(x)}$ and $W^{k,p(x)}$*, Czechoslovak Mathematical J. **41**, 4 (1991), 592–618.
[22] D. MAKHARADZE, A. MESKHI AND M. A. RAGUSA, *Regularity results in grand variable exponent Morrey spaces and applications*, Electron. Res. Arch. **33**, 5 (2025), 2800–2814.
[23] J. MUSIELAK, *Orlicz spaces and modular spaces*, Lecture Notes in Math. **1034**, Springer-Verlag, Berlin, Heidelberg, New York, (1983).
[24] J. MUSIELAK AND W. ORLICZ, *On modular spaces*, Stud. Math. **18**, 1 (1959), 49–65.
[25] H. NAKANO, *Modulared semi-ordered linear spaces*, Maruzen, Co., Ltd., Tokyo, (1950).
[26] H. NAKANO, *Topology and topological linear spaces*, Maruzen, Co., Ltd., Tokyo, (1951).
[27] W. ORLICZ, *Über konjugierte exponentenfolgen*, Stud. Math. **3**, 1 (1931), 200–212.
[28] A. OUAZIZ AND A. ABERQI, *Singular fractional double-phase problems with variable exponent via Morse's theory*, Filomat **38**, 21 (2024), 7579–7595.
[29] RŮŽIČKA, *Electrorheological fluids: Modeling and mathematical theory*, Lecture Notes in Math. **1748**, Springer-Verlag, Berlin, Heidelberg, New York, (2000).
[30] M. RŮŽIČKA AND L. DIENING, *Non-Newtonian fluids and function spaces*, in NAFSA 8-Nonlinear analysis, function spaces and applications. 8, Czech. Acad. Sci., Prague, (2007), 95–143.
[31] I. I. SHARAPUDINOV, *On a topology of the space $L^{p(t)}([0,1])$*, Math. Notes **26**, 4 (1979), 796–806.
[32] V. V. ZHIKOV, *Meyer-type estimates for solving the nonlinear Stokes system*, Differ. Uravn. **33**, 1 (1997), 107–114.
[33] V. V. ZHIKOV, *On some variational problems*, Russian J. Math. Phys. **5**, 1 (1998), 105–116.