

## GENERALIZED CLASS OF BIVARIATE HERMITE AND LAGUERRE POLYNOMIALS

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*Abstract.* This paper introduces and studies a new class of bivariate Hermite-Laguerre polynomials that unify and extend several well-known families of two-dimensional Hermite and Laguerre polynomials. We establish their fundamental properties and demonstrate that these polynomials provide explicit solutions to both fractional and classical heat equations in one and two dimensions, highlighting their analytical richness and potential for diverse applications.

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