

FEKETE-SZEGŐ AND TOEPLITZ TYPE RESULTS FOR q -CONVEX FUNCTION ASSOCIATED WITH MODIFIED q -SIGMOID FUNCTION

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Abstract. In this paper, we introduce a new class of q -convex functions that generalizes the classical convex functions by incorporating q -calculus and subordination to the modified q -sigmoid function. The primary aim is to estimate sharp bounds for the initial coefficients, the Fekete-Szegő functional, coefficient differences, and certain orders of the Toeplitz determinant associated with the defined class.

Mathematics subject classification (2020): 30C45, 30C50.

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