

CORE INCLUSION THEOREM FOR DOUBLE SEQUENCES

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Abstract. This article characterizes when four-dimensional matrices preserve generalized core inclusions for double sequences. Using the disk characterization of Pringsheim cores, this work establishes that a four-dimensional matrix A transforming bounded double sequences into bounded double sequences satisfies $P-C\{A(x)\} \subseteq P-C^{(\alpha)}\{x\}$ for all bounded double sequences if and only if A is RH-regular and $P\text{-}\limsup_{m,n} \sum_{k,l=1,1}^{\infty,\infty} |a_{m,n,k,l}| \leq \alpha$, which is a multidimensional extension of Natarajan's core inclusion theorem, with the special case $\alpha = 1$ providing a double sequence analog of Knopp's core theorem.

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