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ON APPROXIMATION OF GENERALIZED EULER MEANS AND RELATED COMPACT OPERATOR CHARACTERIZATIONS

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Abstract. Upon several investigations on difference sequence spaces, the current article aims to propose a new family of generalized Euler difference sequence spaces with the help of generalized Euler mean and fractional binomial difference operator. Also, this paper discusses certain theorems related to matrix characterizations, operator norms and Hausdorff measure of noncompactness of the new spaces. In addition, the compact operator characterization for the proposed spaces of the binomial difference via the Euler mean has been demonstrated. Moreover, towards the final section, we have proposed the Korovkin type approximation theorem for the proposed operator along with some convergence analysis via modulus of smoothness of order one.

Mathematics subject classification (2020): 46A45, 46A35, 46B45.

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