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GEOMETRIC AND MONOTONIC PROPERTIES OF THE BESSEL FUNCTIONS

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Abstract. In this paper, we obtain sufficient conditions for the parameters of the generalized Bessel functions to be univalent, convex, and starlike in the open unit disk. We also establish an inclusion relation and observe that the main results improve certain existing results.

Mathematics subject classification (2020): 33C10, 30C45.
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