

AN OPERATIONAL AND ASYMPTOTIC THEORY FOR
DEGENERATE LAURICELLA HYPERGEOMETRIC FUNCTIONS

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Abstract. This paper develops an operational and coefficientwise small-parameter theory for degenerate Lauricella hypergeometric functions of types A , B , C , and D . The construction is based on degenerate Pochhammer symbols associated with the degenerate gamma function. We prove first-order, higher-order, and mixed derivative formulas, and show that the deformed families preserve the parameter-shift structure of the classical Lauricella theory. We also obtain generating relations, diagonal reductions, and Euler-type integral representations based on a degenerate binomial kernel. The fixed-deformation case gives entire functions of the variables, while the small-parameter regime connects the deformed coefficients with their classical counterparts. In particular, we derive explicit first-order coefficient expansions as $\lambda \rightarrow 0^+$ and identify the corresponding correction series. Numerical examples in two variables illustrate how the deformation parameter changes the corresponding profiles in numerically stable compact regions.

Mathematics subject classification (2020): 33C65, 33C20, 33E20, 44A10.

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