

CERTAIN GEOMETRIC PROPERTIES OF GENERALIZED k -BESSEL FUNCTIONS

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Abstract. In this paper our aim is to establish some geometric properties (like starlikeness, convexity and close-to-convexity) for the generalized k -Bessel functions. In order to prove our main results, we use some inequalities for ratio of these functions in normalized form and classical result of Fejer [15].

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