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TOWARDS A WELL–DEFINED MEDIAN

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Abstract. The diagonal Δ of $\mathbb{R}^n$ is Chebeshev with respect to the p-norm for every $p \in (1,\infty]$ but not for $p = 1$. As a result, the median is multi-valued, since the median of a data set $\{a_1,\cdots,a_n\}$ can be thought of as the number(s) μ for which the point $(\mu,\cdots,\mu)$ is a point on Δ that best approximates the point $(a_1,\cdots,a_n)$ with respect to the ℓ_1-norm. In this note, it is proved that if $(\mu_p,\cdots,\mu_p)$ is the unique point on Δ that best approximates a fixed point $(a_1,\cdots,a_n)$ with respect to the ℓ_p-norm for $p \in (1,\infty]$, then as p decreases to 1, μ_p converges, and its limit is proposed to be called *the* median of $\{a_1,\cdots,a_n\}$. Along the way, μ_p is shown to be continuous in p for all $p \in (1,\infty]$ in the sense that μ_p converges to μ_q as p goes to q for every $q \in (0,\infty]$.

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