NORM INEQUALITIES FOR THE CHAOTICALLY GEOMETRIC MEAN AND ITS REVERSE

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Abstract. Let $A$ and $B$ be strictly positive operators on a Hilbert space $H$ such that $0 < m \leq B \leq M$ for some scalars $0 < m < M$ and $h_B = \frac{M}{m}$. We prove a norm inequality for the chaotically geometric mean $A \diamond_\alpha B = e^{(1-\alpha) \log A + \alpha \log B}$ and its reverse: For each real number $\alpha \in \mathbb{R}$

$$S(h_B)^{-1} \left\| A^{1-\alpha} B^\alpha A^{\frac{1}{2}} \right\| \leq \left\| A \diamond_\alpha B \right\| \leq \left\| A^{1-\alpha} B^\alpha A^{\frac{1}{2}} \right\|$$

where the constant $S(h)$ is the Specht ratio and $\| \cdot \|$ is the operator norm.


Keywords and phrases: Positive operator, Araki’s inequality, chaotically geometric mean, Specht ratio, a generalized Kantorovich constant, reverse inequality.

REFERENCES