

NOTES ON CERTAIN INEQUALITIES BY HÖLDER, LEWENT AND KY FAN

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Abstract. The aim of this paper is twofold. First we show that the famous Hölder inequality (which one, according to [10] should be called as the Rogers inequality) was discovered by Grolous [5] and Besso [3] about 10 years before Rogers. On the other hand, a result obtained by Lewent [9] leads to a new proof of the famous Ky Fan inequality [2]. Related results are pointed out, too.

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