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STABILITY OF HOMOMORPHISMS ON JB^* –TRIPLES ASSOCIATED TO A CAUCHY–JENSEN TYPE FUNCTIONAL EQUATION

ABBAS NAJATI

Abstract. In this paper, we investigate homomorphisms between JB^* -triples, and derivations on JB^* -triples associated to the following Cauchy–Jensen type additive functional equation

$$f\left(\frac{x+y}{2}+z\right)+f\left(\frac{x+z}{2}+y\right)+f\left(\frac{y+z}{2}+x\right)=2[f(x)+f(y)+f(z)].$$

The concept of Hyers-Ulam-Rassias stability originated from Th. M. Rassias' stability theorem that appeared in his paper: On the stability of the linear mapping in Banach spaces, Proc. Amer. Math. Soc. **72** (1978), 297–300.

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