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AN EXTENSION ON THE HARDY–HILBERT INTEGRAL INEQUALITY AND ITS APPLICATIONS

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Abstract. In this article, it is shown that some new extensions on the Hardy-Hilbert inequality related to exponent function can be established by introducing a parameter λ $(1-\frac{q}{p}<\lambda\leqslant 2$, $\frac{1}{p}+\frac{1}{q}=1$ and $p\geqslant q>1)$ and two exponent functions Aa^x $(A>0, a>1)$ and Bb^y $(B>0, b>1)$. In particular, for the case $p=2$, an extension of the Hilbert integral inequality is built. As an application, a new Hardy-Littlewood integral inequality is given.

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