Journal of Mathematical Inequalities
Volume 1, Number 2 (2007), 225–241

A VISCOSITY RELAXED–EXTRAGRADIENT METHOD FOR MONOTONE VARIATIONAL INEQUALITIES AND FIXED POINT PROBLEMS

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Abstract. In this paper we introduce a viscosity relaxed-extragradient method for finding a common element of the set of fixed points of a nonexpansive mapping and the set of solutions of the variational inequality problem for a monotone, Lipschitz-continuous mapping in a real Hilbert space H. The viscosity relaxed-extragradient method is based on two methods: extragradient-like approximation method and viscosity approximation method. We derive a weak convergence theorem for two sequences generated by this method. Utilizing this theorem we also construct an iterative process for finding a common zero of two mappings, one of which is a monotone, Lipschitz continuous mapping of H into itself and the other taken from the more general class of maximal monotone mappings of H into 2^H.

Mathematics subject classification (2000): 47H09, 47J20.
Key words and phrases: Variational inequality, nonexpansive mapping, extragradient-like approximation method, viscosity approximation method, monotone mapping, fixed point, weak convergence, demiclosedness principle, Opial's condition.

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