

IMPLICIT ITERATION SCHEME WITH PERTURBED MAPPING FOR COMMON FIXED POINTS OF A FINITE FAMILY OF LIPSCHITZ PSEUDOCONTRACTIVE MAPPINGS

LU-CHUAN CENG¹, ADRIAN PETRUŞEL AND JEN-CHIH YAO²

Abstract. Let E be a real Banach space, $\{T_i\}_{i=1}^N$ be a finite family of continuous pseudocontractive self mappings of E and $G : E \rightarrow E$ be a mapping which is both δ -strongly accretive and λ -strictly pseudocontractive of Browder-Petryshyn type such that $\delta + \lambda \geq 1$. We propose a new implicit iteration scheme with perturbed mapping G for the approximation of common fixed points of $\{T_i\}_{i=1}^N$. For an arbitrary initial point $x_0 \in E$, the sequence $\{x_n\}_{n=1}^\infty$ is defined by

$$x_n = \alpha_n(x_{n-1} - \lambda_n G(x_{n-1})) + (1 - \alpha_n)T_n x_n$$

where $T_n = T_{n \bmod N}$, $\{\alpha_n\}_{n=1}^\infty \subset [a, b] \subset]0, 1[$ and $\{\lambda_n\}_{n=1}^\infty \subset [0, 1[$. We establish some weak convergence theorems for this implicit iteration scheme. Also, necessary and sufficient conditions for strong convergence of this implicit iteration scheme are obtained.

Mathematics subject classification (2000): 47H09, 47H10, 47H17.

Key words and phrases: Continuous pseudocontractive mapping, Implicit iteration process with perturbed mapping, Strictly pseudocontractive mapping, Common fixed point, Demiclosedness principle.

REFERENCES

- [1] M. O. OSILIKE, *Implicit iteration process for common fixed points of a finite family of strictly pseudocontractive maps*, J. Math. Anal. Appl. 294 (2004) 73-81.
- [2] M. O. OSILIKE, A. UDOMENE, *Demiclosedness principle and convergence results for strictly pseudocontractive mappings of Browder-Petryshyn type*, J. Math. Anal. Appl. 256 (2001) 431-445.
- [3] H. K. XU, R. G. ORI, *An implicit iteration process for nonexpansive mappings*, Numer. Funct. Anal. Optim. 22 (2001) 767-773.
- [4] S. S. CHANG, Y. J. CHO, H. ZHOU, *Iterative Methods for Nonlinear Operator Equations in Banach Spaces*, Nova Science Publishers, New York, 2002.
- [5] R. D. CHEN, Y. S. SONG, H. Y. ZHOU, *Convergence theorems for implicit iteration process for a finite family of continuous pseudocontractive mappings*, J. Math. Anal. Appl. 314 (2006) 701-709.
- [6] L. C. ZENG, J. C. YAO, *Implicit iteration scheme with perturbed mapping for common fixed points of a finite family of nonexpansive mappings*, Nonlinear Anal. TMA 64 (2006) 2507-2515.
- [7] M. O. OSILIKE, S. C. ANIAGBOSOR, B. G. AKUCHU, *Fixed points of asymptotically demicontractive mappings in arbitrary Banach spaces*, PanAmer. Math. J. 12 (2002) 77-88.
- [8] K. K. TAN, H. K. XU, *Approximating fixed points of nonexpansive mappings by the Ishikawa iteration process*, J. Math. Anal. Appl. 178 (1993) 301-308.
- [9] Z. OPIAL, *Weak convergence of the sequence of successive approximations for nonexpansive mappings*, Bull. Amer. Math. Soc. 73 (1967) 591-597.
- [10] K. DEIMLING, *Nonlinear Functional Analysis*, Springer-Verlag New York 1985.
- [11] H. ZHOU, *Convergence theorems of common fixed points for a finite family of Lipschitz pseudocontractions in Banach spaces*, Nonlinear Anal. TMA (2007) doi:10.1016/j.na.2007.02.041

- [12] R. E. BRUCK, *A simple proof of the mean ergodic theorem for nonlinear contractions in Banach spaces*, Israel J. Math. 32 (1979) 107-116.
- [13] W. TAKAHASHI, *Nonlinear Functional Analysis-Fixed Point Theory and Applications*, Yokohama Publishers Inc. Yokohama 2000.
- [14] K. K. TAN, H. K. XU, *Fixed point iteration process for asymptotically nonexpansive mappings*, Proc. Amer. Math. Soc. 122 (1994) 733-739.
- [15] K. DEIMLING, *Zeros of accretive operators*, Manuscripta Math. 13(1974) 365-374.