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VECTOR VARIATIONAL–LIKE INEQUALITIES WITH RELAXED η-α PSEUDOMONOTONE MAPPINGS IN BANACH SPACES

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Abstract. In this paper, we introduce two new concepts of relaxed η-α pseudomonotonicity and relaxed η-α demipseudomonotonicity as well as two classes of vector variational-like inequalities with relaxed η-α pseudomonotone mappings and relaxed η-α demipseudomonotone mappings in Banach spaces. By using KKM technique, we obtain the existence of solutions for vector variational-like inequalities with relaxed η-α pseudomonotone mappings in reflexive Banach spaces. We also show the solvability of vector variational-like inequalities with relaxed η-α demipseudomonotone mappings in reflexive Banach spaces by means of Kakutani-Fan-Glicksberg fixed-point theorem. The results presented in this paper extend and improve some corresponding results of several authors.

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