

ON THE GENERALIZED HYERS-ULAM STABILITY OF SWIATAK'S FUNCTIONAL EQUATION

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Abstract. In this paper we shall study the generalized Hyers-Ulam stability of Swiatk's functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) + g(x)g(y), \quad x, y \in G,$$

where G is an abelian group and $f, g : G \rightarrow \mathbb{C}$ are complex-valued functions satisfying the condition $g(e) \neq 0$.

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