

SPECTRAL PROPERTIES OF A CLASS OF SINGULAR DIFFERENTIAL OPERATORS

A. A. KALYBAY, R. OINAROV AND L.-E. PERSSON

Abstract. We consider the operator

$$A_0 f = (-1)^n \frac{1}{v(t)} \left(D_\rho^n \right)^* \left[u^2(t) D_\rho^n \left(\frac{f(t)}{v(t)} \right) \right],$$

where

$$D_\rho^n f(t) = \frac{d^k}{dt^k} \left[\rho(t) \frac{d^m f(t)}{dt^m} \right], \quad \left(D_\rho^n \right)^* f(t) = \frac{d^m}{dt^m} \left[\rho(t) \frac{d^k f(t)}{dt^k} \right], \quad k + m = n.$$

Our main aim is to prove some spectral properties of a natural extension of this operator. In order to prove this we need to prove some properties of a function space, connected to the operator D_ρ^n , and some embedding theorems of independent interest.

Mathematics subject classification (2000): 46E35, 47B25.

Key words and phrases: Inequalities, weights, weighted function space, weighted differential operator, spectrum, eigenvalues, spectrum discreteness, spectrum positive definiteness, resolvent nuclearity.

REFERENCES

- [1] APYSHEV, O.,D., AND OTELBAYEV, M., *On spectrum of one class of differential operators and some embedding theorems*, Izv. Acad. Sci. SSSR, Ser. Mat., **43** (1979), 739–764.
- [2] Birman, M. S., *On spectrum of singular boundary problems*, Matem. Sbornik, 55 (97) : 2 (1961), 125–173.
- [3] Glazman, I. M., *Direct Methods of Qualitative Spectral Analysis of Singular Differential Operators*, Moscow, Physmatgiz, 1963.
- [4] Kalybay, A. A., *A Generalization of the weighted Hardy inequality for one class of integral operators*, Siberian Math. J. 45 (2004), iss. 1, 100–112.
- [5] Kalybay, A. A., and Persson, L.-E., *Three weights higher order Hardy inequalities*, Function Spaces and Applications, vol. 4, 2 (2006), 163–191.
- [6] Kudryavcev, L. D., *On equivalent norms in weighted spaces*, Trudy Mat. Inst. Steklov, 170 (1984), 161–190.
- [7] Kufner, A. and Persson, L.-E., *Weighted Inequalities of Hardy Type*. World Scientific, New Jersey/London/Singapore/Hong Kong, 2003, 357 pages.
- [8] Mynbayev, K., and Otelbayev, M., *Weighted functional spaces and differential operator spectrum*, Moscow, Nauka, 1988, 288 pp.
- [9] Muckenhoupt, B., *Hardy’s inequality with weights*, Stud. Math., vol. XLIV, 1 (1972), 31–38.
- [10] Oinarov, R., *Boundedness and compactness of superposition of fractional integration operators and their applications*, Proc. Function Spaces, Differential Operators and Nonlinear Analysis (FSDONA 2004), Math. Institute, Acad. Sci., Czech Republic, (2004), 213–235.
- [11] Otelbayev, M., *Criteria of spectrum discreteness of one singular operator and some embedding theorems*, Differential Equations, 13 (1977), N 1, 111–120.

- [12] Otelbayev, M., Criteria of resolvent kernelness of Sturm – Liouville operator, *Mat. Zametki*, 25 (1979), N 4, 569–572.
- [13] Otelbayev, M., Estimates of spectrum of Sturm – Liouville operator, Almaty, Gylym, 1990, 192 pp.
- [14] Riss, F., and Sekefal'vi–Nad', B., Lectures on Functional Analysis, Moscow, Mir, 1979, 589 pp.
- [15] Stepanov, V. D., Weighted inequalities of Hardy type for higher–order derivatives and their applications, *Dokl. Akad. Nauk SSSR*, 302 (1988), N1, 1059–1062; transl. in *Soviet Math. Dokl.* 38 (1989), N2, 389–393.
- [16] Stepanov, V. D., Two–weighted estimates of Riemann–Liouville integrals, *Izv. Akad. Nauk SSSR, Ser. Mat.*, 54 (1990), N3, 645–654; transl. in *Math. USSR-Izv.*, 36 (1991), 669–681.