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HYERS–ULAM STABILITY OF THE FIRST ORDER LINEAR DIFFERENTIAL EQUATION FOR BANACH SPACE–VALUED HOLOMORPHIC MAPPINGS

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Abstract. Let Ω be a convex open set of $\mathbb{C}$, and let X be a complex Banach space. Suppose that $p\colon \Omega \to \mathbb{C}$ and $q\colon \Omega \to X$ are holomorphic. We give sufficient conditions in order that the first order linear differential equation $f'(z) + p(z)f(z) + q(z) = 0$ for X-valued holomorphic mapping $f : \Omega \to X$ has the Hyers-Ulam stability.

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