

## AN EXTENDED MATRIX EXPONENTIAL FORMULA

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*Abstract.* In this paper we present matrix exponential formulae for the geometric and spectral geometric means of positive definite matrices using a conjectured exponential formula that solved by Wasin So [Linear Algebra Appl. 379 (2004)].

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*Key words and phrases:* Exponential formula, Hermitian matrix, positive definite matrix, geometric and spectral mean, Lie-Trotter formula.

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