

MEANINGFUL INEQUALITIES

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Abstract. The fundamental Theorem on Means serves to “explain” many recently published inequalities, and even to suggest new versions. Our aim here is to explore the possibilities.

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