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HARDY AND RELLICH INEQUALITIES WITH REMAINDERS

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Abstract. In this paper our primary concern is with the establishment of weighted Hardy inequalities in $L^p(\Omega)$ and Rellich inequalities in $L^2(\Omega)$ depending upon the distance to the boundary of domains $\Omega \subset \mathbb{R}^n$ with a finite diameter $D(\Omega)$. Improved constants are presented in most cases.

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