Journal of Mathematical Inequalities
Volume 1, Number 4 (2007), 491–501

CONTRACTIONS OF A NUMERICAL SEMIGROUP

J. C. ROSALES

Abstract. Given a numerical semigroup S, a positive integer a and $m \in S\setminus\{0\}$, we introduce the set $C(S,a,m) = \{x \in \mathbb{N} \mid aw(x \bmod m) \leqslant x\}$, where $\{w(0), w(1), \ldots, w(m-1)\}$ is the Apéry set of m in S, which is a numerical semigroup and that we call (a,m) - contraction of S. We study the Frobenius number and the singularity degree of $C(S,a,m)$. We also characterize the contractions $C(S,a,m)$ that are symmetric and pseudo-symmetric numerical semigroups. Finally we see that the contractions of $\mathbb{N}$ are solutions of modular Diophantine inequalities.

Mathematics subject classification (2000): 20M14, 13H1.
Key words and phrases: Numerical semigroup, symmetric, pseudo-symmetric, modular Diophantine inequality, Frobenius number, singularity degree.

REFERENCES

[1] R. APÉRY, *Sur les branches superlinéaires des courbes algébriques*, C.R. Acad. Sci. Paris 222 (1946), 1198-1200.
[2] V. BARUCCI, D. E. DOBBS AND M. FONTANA, *Maximality Properties in Numerical Semigroups and Applications to One-Dimensional Analytically Irreducible Local Domains*, Memoirs of the Amer. Math. Soc. **598** (1997).
[3] A. BRAUER AND J. E. SCHOCKLEY, *On a problem of Frobenius*, J. Reine Angew. Math. **211** (1962), 215-220.
[4] M. DELGADO, J. C. ROSALES, *Modular Diophantine inequalities and rotations of numerical semigroups*, J. Australian Math. Soc. (to appear).
[5] R. FRÖBERG, G. GOTTLIEB AND R. HÄGGKVIST, *On numerical semigroups*, Semigroup Forum **35** (1987), 63-83.
[6] J. L. RAMÍREZ ALFONSÍN, *The diophantine Frobenius problem*, Oxford Univ. Press, 2005.
[7] J. C. ROSALES, *On symmetric numerical semigroups*, J. Algebra, **182** (1996) 422-434.
[8] J. C. ROSALES AND M. B. BRANCO, *Irreducible numerical semigroups*, Pacific J. Math. **209** (2003), 131-143.
[9] J. C. ROSALES AND P. A. GARCÍA-SÁNCHEZ, *Finitely generated commutative monoids*, Nova Science Publishers, New York, 1999.
[10] J. C. ROSALES, P. A. GARCÍA-SÁNCHEZ, J. M. URBANO-BLANCO, *Modular Diophantine inequalities and numerical semigroups*, Pacific J. Math. **218** (2005), 379-398.
[11] J. C. ROSALES, P. A. GARCÍA-SÁNCHEZ, J. M. URBANO-BLANCO, *Correction to "Modular Diophantine inequalities and numerical semigroups, Pacific J. Math.* **218** *(2005), 379-398"*, Pacific J. Math. **220** (2005),199.
[12] E. S. SELMER, *On a linear diophantine problem of Frobenius*, J. Reine Angew. Math. **293/294** (1977), 1-17.
[13] J. J. SYLVESTER, *Excursus on rational fractions and partitions*, Amer. J. Math. **5** (1882), 119-136.
[14] J. J. SYLVESTER, *Mathematical questions with their solutions*, Educational Times **41** (1884), 21.

Paper JMI-01-41