ON A NEW GENERALIZATION OF MARTINS’ INEQUALITY

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Abstract. Let \( n, m \in \mathbb{N} \) and \( \{a_i\}_{i=1}^{n+m} \) be an increasing, logarithmically concave, positive, and nonconstant sequence such that the sequence \( \{i \left[ \frac{a_{i+1}}{a_i} - 1 \right]\}_{i=1}^{n+m-1} \) is increasing. Then the following inequality between ratios of the power means and of the geometric means holds:

\[
\left( \frac{1}{n} \sum_{i=1}^{n} a_i^r \right)^{1/r} < \frac{\sqrt[n]{a_n!}}{\sqrt[n+m]{a_{n+m}!}},
\]

where \( r \) is a positive number. \( a_n! \) denotes the sequence factorial defined by \( \prod_{i=1}^{n} a_i \). The upper bound is the best possible.


Key words and phrases: Martins’s inequality, Alzer’s inequality, König’s inequality, increasing sequence, logarithmically concave, ratio, power mean, geometric mean.

REFERENCES


