

ON INEQUALITIES FOR SUMS OF BOUNDED RANDOM VARIABLES

IOSIF PINELIS

Abstract. A new upper bound on $P(a_1\eta_1 + a_2\eta_2 + \dots \geq x)$ is obtained, where η_1, η_2, \dots are independent zero-mean random variables such that $|\eta_i| \leq 1$ for all i . A multidimensional analogue of this result and extensions to (super)martingales are presented, as well as an application to self-normalized sums (or, equivalently, to t -statistics).

Mathematics subject classification (2000): 60E15, 60G50, 60G42, 60G48, 26A48, 26D10.

Keywords and phrases: Upper bounds, probability inequalities, bounded random variables, Rademacher random variables, sums of independent random variables, (super)martingales, self-normalized sums, t -statistics.

REFERENCES

- [1] V. BENTKUS, *A remark on the inequalities of Bernstein, Prokhorov, Bennett, Hoeffding, and Talagrand*, Lithuanian Math. J., **42**, (2002), 262–269.
- [2] V. BENTKUS, *An inequality for tail probabilities of martingales with differences bounded from one side*, J. Theoret. Probab., **16**, (2003), 161–173.
- [3] V. BENTKUS, *On Hoeffding's inequalities*, Ann. Probab., **32**, (2004), 1650–1673.
- [4] S. G. BOBKOV, F. GÖTZE, C. HOUDRÉ, *On Gaussian and Bernoulli covariance representations*, Bernoulli, **7**, (2002), 439–451.
- [5] M. L. EATON, *A probability inequality for linear combinations of bounded random variables*, Ann. Statist., **2**, (1974), 609–614.
- [6] M. L. EATON, B. EFRON, *Hotelling's T^2 test under symmetry conditions*, J. Amer. Statist. Assoc., **65**, (1970), 702–711.
- [7] D. EDELMAN, *An inequality of optimal order for the tail probabilities of the T statistic under symmetry*, J. Amer. Statist. Assoc., **85**, (1990), 120–122.
- [8] B. EFRON, *Student's t test under symmetry conditions*, J. Amer. Statist. Assoc., **64**, (1969), 1278–1302.
- [9] S. E. GRAVERSEN, G. PEŠKIR, *Extremal problems in the maximal inequalities of Khintchine*, Math. Proc. Cambridge Philos. Soc., **123**, (1998), 169–177.
- [10] U. HAAGERUP, *The best constants in the Khintchine inequality*, Studia Math., **70**, (1982), 231–283.
- [11] W. HOEFFDING, *Probability inequalities for sums of bounded random variables*, J. Amer. Statist. Assoc., **58**, (1963), 13–30.
- [12] G. A. HUNT, *An inequality in probability theory*, Proc. Amer. Math. Soc., **6**, (1955), 506–510.
- [13] A. KHINCHIN, *Über dyadische Brüche*, Math. Z., **18**, (1923), 109–116.
- [14] G. PEŠKIR, A. N. SHIRYAEV, *The Inequalities of Khintchine and Expanding Sphere of Their Action*, Russian Math. Surveys, **50**, (1995), 849–904.
- [15] I. PINELIS, *Extremal probabilistic problems and Hotelling's T^2 test under symmetry condition*, (1991), Preprint, URL: <http://arxiv.org/abs/math/0701806>.
- [16] I. PINELIS, *Extremal probabilistic problems and Hotelling's T^2 test under a symmetry condition*, Ann. Statist., **22**, (1994), 357–368.
- [17] I. PINELIS, *Optimal tail comparison based on comparison of moments*, in High dimensional probability (Oberwolfach, 1996), Progr. Probab., Birkhäuser, Basel, **43**, (1998), 297–314.

- [18] I. PINELIS, *Fractional sums and integrals of r -concave tails and applications to comparison probability inequalities*, in Advances in stochastic inequalities (Atlanta, GA, 1997), Contemp. Math., Amer. Math. Soc., Providence, RI, **234**, (1999), 149–168.
- [19] I. PINELIS, *On exact maximal Khinchine inequalities*, in High dimensional probability, II (Seattle, WA, 1999), Progr. Probab., Birkhäuser Boston, Boston, MA, **47**, (2000), 49–63.
- [20] I. PINELIS, *L'Hospital type rules for oscillation, with applications*, J. Inequal. Pure Appl. Math., **2**, (2001), no. 3, Article 33, 24 pp. (electronic).
- [21] I. PINELIS, *Monotonicity properties of the relative error of a Padé approximation for Mills' ratio*, J. Inequal. Pure Appl. Math., **3**, (2002), no. 2, Art. 20, 8 pp. (electronic).
- [22] I. PINELIS, *L'Hospital type rules for monotonicity: applications to probability inequalities for sums of bounded random variables*, J. Inequal. Pure Appl. Math., **3**, (2002), no. 1, Article 7, 9 pp. (electronic).
- [23] I. PINELIS, *Dimensionality reduction in extremal problems for moments of linear combinations of vectors with random coefficients*, in Stochastic inequalities and applications, Progr. Probab., Birkhäuser, Basel, **56**, (2003), 169–185.
- [24] I. PINELIS, *Binomial upper bounds on generalized moments and tail probabilities of (super)martingales with differences bounded from above*, in IMS Lecture Notes-Monograph Series, High Dimensional Probability, Institute of Mathematical Statistics, **51**, (2006). DOI: 10.1214/074921706000000743. URL: <http://arxiv.org/abs/math.PR/0512301>.
- [25] I. PINELIS, *On normal domination of (super)martingales*, Electronic J. Probab., **11**, (2006), Paper 39, 1049–1070. URL: <http://www.math.washington.edu/~ejpecp/viewarticle.php?id=1648&layout=abstract>.
- [26] I. PINELIS, *On l'Hospital-type rules for monotonicity*, J. Inequal. Pure Appl. Math., **7**, (2006), no. 2, Article 40 (electronic).
- [27] I. PINELIS, (2006). *Exact inequalities for sums of asymmetric random variables, with applications*, Probab. Theory Related Fields **139**, (2007), no. 3-4, 605–635.
- [28] I. PINELIS, (2007). *Toward the best constant factor for the Rademacher-Gaussian tail comparison*, ESAIM Probab. Stat., **11**, (2007), 412–426 (electronic).
- [29] P. WHITTLE, *Bounds for the moments of linear and quadratic forms in independent variables*, Teor. Verojatnost. i Primenen., **5**, (1960), 331–335.